APPENDIX A Basic Mathematics Review

Preview

PREVIEW

This appendix reviews some of the basic math skills that are necessary for the statistical calculations presented in this book. Many students already will know some or all of this material. Others will need to do extensive work and review. To help you assess your own skills, we include a skills assessment exam here. You should allow approximately 30 minutes to complete the test. When you finish, grade your test using the answer key on page 569.

Notice that the test is divided into five sections. If you miss more than three questions in any section of the test, you probably need help in that area. Turn to the section of this appendix that corresponds to your problem area. In each secti you will find a general review, examples, and additional practice problems. After reviewing th appropriate section and doing the practice probl turn to the end of the appendix. You will find an version of the skills assessment exam. If you stil more than three questions in any section of the e continue studying. Get assistance from an instruc or a tutor if necessary. At the end of this appendi list of recommended books for individuals who n a more extensive review than can be provided he We stress that mastering this material now will m the rest of the course much easier.

SKILLS ASSESSMENT PREVIEW EXAM

SECTION 1

(corresponding to Section A.1 of this appendix)

1. $3 + 2 \times 7 = ?$
2. $(3 + 2) \times 7 = ?$
3. $3 + 2^2 - 1 = ?$
4. $(3 + 2)^2 - 1 = ?$
5. $12/4 + 2 = ?$
6. $12/(4 + 2) = ?$
7. $12/(4 + 2)^2 = ?$
8. $2 \times (8 - 2^2) = ?$
9. $2 \times (8 - 2)^2 = ?$
10. $3 \times 2 + 8 - 1 \times 6 = ?$
11. $3 \times (2 + 8) - 1 \times 6 = ?$
12. $3 \times 2 + (8 - 1) \times 6 = ?$

SECTION 2

(corresponding to Section A.2 of this appendix)

1. The fraction $\frac{3}{4}$ corresponds to a percentage of ________.
2. Express 30% as a fraction.
3. Convert $\frac{12}{40}$ to a decimal.
4. $\frac{2}{13} + \frac{8}{13} = ?$
5. $1.375 + 0.25 = ?$
6. $\frac{2}{5} \times \frac{1}{4} = ?$
7. $\frac{1}{8} + \frac{2}{3} = ?$
8. $3.5 \times 0.4 = ?$
9. $\frac{1}{5} \div \frac{3}{4} = ?$
10. $3.75/0.5 = ?$
11. In a group of 80 students, 20% are psychology majors. How many psychology majors are in this group?
12. A company reports that two-fifths of its employees are women. If there are 90 employees, how many are women?

SECTION 3

(corresponding to Section A.3 of this appendix)

1. $3 + (-2) + (-1) + 4 = ?$
2. $6 - (-2) = ?$
3. $-2 - (-4) = ?$
4. $6 + (-1) - 3 - (-2) - (-5) = ?$
5. $4 \times (-3) = ?$
6. $-2 \times (-6) = ?$
7. $-3 \times 5 = ?$
8. $-2 \times (-4) \times (-3) = ?$
9. $12 \div (-3) = ?$
10. $-18 \div (-6) = ?$
11. $-16 \div 8 = ?$
12. $-100 \div (-4) = ?$

SECTION 4

(corresponding to Section A.4 of this appendix)

For each equation, find the value of *X*.

1. $X + 6 = 13$
2. $X - 14 = 15$
3. $5 = X - 4$
4. $3X = 12$
5. $72 = 3X$
6. $X/5 = 3$
7. $10 = X/8$
8. $3X + 5 = -4$
9. $24 = 2X + 2$
10. $(X + 3)/2 = 14$
11. $(X - 5)/3 = 2$
12. $17 = 4X - 11$

SECTION 5

(corresponding to Section A.5 of this appendix)

1. $4^3 = ?$
2. $\sqrt{25 - 9} = ?$
3. If $X = 2$ and $Y = 3$, then $XY^3 = ?$
4. If $X = 2$ and $Y = 3$, then $(X + Y)^2 = ?$
5. If $a = 3$ and $b = 2$, then $a^2 + b^2 = ?$
6. $(-3)^3 = ?$
7. $(-4)^4 = ?$
8. $\sqrt{4} \times 4 = ?$
9. $36/\sqrt{9} = ?$
10. $(9 + 2)2 = ?$
11. $5^2 + 2^3 = ?$
12. If $a = 3$ and $b = -1$, then $a^2b^3 = ?$

The answers to the skills assessment exam are at the end of the appendix (pages 569–570).

A.1 SYMBOLS AND NOTATION

Table A.1 presents the basic mathematical symbols that you should know, along examples of their use. Statistical symbols and notation are introduced and expla throughout this book as they are needed. Notation for exponents and square roo covered separately at the end of this appendix.

Parentheses are a useful notation because they specify and control the order of c putations. Everything inside the parentheses is calculated first. For example,

$$(5 + 3) \times 2 = 8 \times 2 = 16$$

Changing the placement of the parentheses also changes the order of calculati For example,

$$5 + (3 \times 2) = 5 + 6 = 11$$

ORDER OF OPERATIONS

Often a formula or a mathematical expression will involve several different arithn operations, such as adding, multiplying, squaring, and so on. When you encou these situations, you must perform the different operations in the correct seque Following is a list of mathematical operations, showing the order in which they a be performed.

1. Any calculation contained within parentheses is done first.
2. Squaring (or raising to other exponents) is done second.
3. Multiplying and/or dividing is done third. A series of multiplication and/or d sion operations should be done in order from left to right.
4. Adding and/or subtracting is done fourth.

The following examples demonstrate how this sequence of operations is applie different situations.

To evaluate the expression

$$(3 + 1)^2 - 4 \times 7/2$$

first, perform the calculation within parentheses:

$$(4)^2 - 4 \times 7/2$$

Next, square the value as indicated:

$$16 - 4 \times 7/2$$

TABLE A.1

Symbol	*Meaning*	*Example*
+	Addition	$5 + 7 = 12$
−	Subtraction	$8 - 3 = 5$
×, ( )	Multiplication	$3 \times 9 = 27$, $3(9) = 27$
÷, /	Division	$15 \div 3 = 5$, $15/3 = 5$, $\frac{15}{3} = 5$
>	Greater than	$20 > 10$
<	Less than	$7 < 11$
≠	Not equal to	$5 \neq 6$

Then perform the multiplication and division:

$$16 - 14$$

Finally, do the subtraction:

$$16 - 14 = 2$$

A sequence of operations involving multiplication and division should be performed in order from left to right. For example, to compute $12/2 \times 3$, you divide 12 by 2 and then multiply the result by 3:

$$12/2 \times 3 = 6 \times 3 = 18$$

Notice that violating the left-to-right sequence can change the result. For this example, if you multiply before dividing, you will obtain

$$12/2 \times 3 = 12/6 = 2 \qquad \text{(This is wrong.)}$$

A sequence of operations involving only addition and subtraction can be performed in any order. For example, to compute $3 + 8 - 5$, you can add 3 and 8 and then subtract 5:

$$(3 + 8) - 5 = 11 - 5 = 6$$

or you can subtract 5 from 8 and then add the result to 3:

$$3 + (8 - 5) = 3 + 3 = 6$$

A mathematical expression or formula is simply a concise way to write a set of instructions. When you evaluate an expression by performing the calculation, you simply follow the instructions. For example, assume you are given these instructions:

1. First, add 3 and 8.
2. Next, square the result.
3. Next, multiply the resulting value by 6.
4. Finally, subtract 50 from the value you have obtained.

You can write these instructions as a mathematical expression.

1. The first step involves addition. Because addition is normally done last, use parentheses to give this operation priority in the sequence of calculations:

   $(3 + 8)$

2. The instruction to square a value is noted by using the exponent 2 beside the value to be squared:

   $(3 + 8)^2$

3. Because squaring has priority over multiplication, you can simply introduce the multiplication into the expression:

   $6 \times (3 + 8)^2$

4. Addition and subtraction are done last, so simply write in the requeste subtraction:

$$6 \times (3 + 8)^2 - 50$$

To calculate the value of the expression, you work through the sequence tions in the proper order:

$$6 \times (3 + 8)^2 - 50 = 6 \times (11)^2 - 50$$
$$= 6 \times (121) - 50$$
$$= 726 - 50$$
$$= 676$$

As a final note, you should realize that the operation of squaring (or raisin exponent) applies only to the value that immediately precedes the exponent. For e

$2 \times 3^2 = 2 \times 9 = 18$ (Only the 3 is squared.)

If the instructions require multiplying values and then squaring the produ must use parentheses to give the multiplication priority over squaring. For exa multiply 2 times 3 and then square the product, you would write

$$(2 \times 3)^2 = (6)^2 = 36$$

LEARNING CHECK

1. Evaluate each of the following expressions:
   a. $4 \times 8/2^2$
   b. $4 \times (8/2)^2$
   c. $100 - 3 \times 12/(6 - 4)^2$
   d. $(4 + 6) \times (3 - 1)^2$
   e. $(8 - 2)/(9 - 8)^2$
   f. $6 + (4 - 1)^2 - 3 \times 4^2$
   g. $4 \times (8 - 3) + 8 - 3$

ANSWERS 1. a. 8 b. 64 c. 91 d. 40 e. 6 f. −33 g. 25

A.2 PROPORTIONS: FRACTIONS, DECIMALS, AND PERCENTAGE

A proportion is a part of a whole and can be expressed as a fraction, a decima percentage. For example, in a class of 40 students, only 3 failed the final exam. The proportion of the class that failed can be expressed as a fraction

$$\text{fraction} = \frac{3}{40}$$

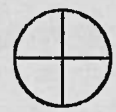
or as a decimal value

$$\text{decimal} = 0.075$$

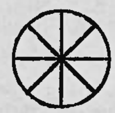
or as a percentage

$$\text{percentage} = 7.5\%$$

In a fraction, such as $\frac{3}{4}$, the bottom value (the denominator) indicates the number of equal pieces into which the whole is split. Here the "pie" is split into 4 equal pieces:

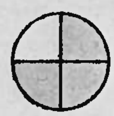


If the denominator has a larger value—say, 8—then each piece of the whole pie is smaller:



A larger denominator indicates a smaller fraction of the whole.

The value on top of the fraction (the numerator) indicates how many pieces of the whole are being considered. Thus, the fraction $\frac{3}{4}$ indicates that the whole is split evenly into 4 pieces and that 3 of them are being used:



A fraction is simply a concise way of stating a proportion: "Three out of four" is equivalent to $\frac{3}{4}$. To convert the fraction to a decimal, you divide the numerator by the denominator:

$$\frac{3}{4} = 3 \div 4 = 0.75$$

To convert the decimal to a percentage, simply multiply by 100, and place a percent sign (%) after the answer:

$$0.75 \times 100 = 75\%$$

The U.S. money system is a convenient way of illustrating the relationship between fractions and decimals. "One quarter," for example, is one-fourth $\left(\frac{1}{4}\right)$ of a dollar, and its decimal equivalent is 0.25. Other familiar equivalencies are as follows:

	Dime	*Quarter*	*50 Cents*	*75 Cents*
Fraction	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Decimal	0.10	0.25	0.50	0.75
Percentage	10%	25%	50%	75%

FRACTIONS

1. **Finding Equivalent Fractions.** The same proportional value can be expressed by many equivalent fractions. For example,

$$\frac{1}{2}=\frac{2}{4}=\frac{10}{20}=\frac{50}{100}$$

To create equivalent fractions, you can multiply the numerator and denominator by the same value. As long as both the numerator and the denominator of the fraction are multiplied by the same value, the new fraction will be equivalent to the original. For example,

$$\frac{3}{10}=\frac{9}{30}$$

because both the numerator and the denominator of the original fraction have been multiplied by 3. Dividing the numerator and denominator of a fraction by the same value will also result in an equivalent fraction. By using division, you can reduce a fraction to a simpler form. For example,

$$\frac{40}{100}=\frac{2}{5}$$

because both the numerator and the denominator of the original fraction have been divided by 20.

You can use these rules to find specific equivalent fractions. For example, find the fraction that has a denominator of 100 and is equivalent to $\frac{3}{4}$. That is,

$$\frac{3}{4}=\frac{?}{100}$$

Notice that the denominator of the original fraction must be multiplied by 25 to produce the denominator of the desired fraction. For the two fractions to be equal, both the numerator and the denominator must be multiplied by the same number. Therefore, we also multiply the top of the original fraction by 25 and obtain

$$\frac{3\times 25}{4\times 25}=\frac{75}{100}$$

2. **Multiplying Fractions.** To multiply two fractions, you first multiply the numerators and then multiply the denominators. For example,

$$\frac{3}{4}\times\frac{5}{7}=\frac{3\times 5}{4\times 7}=\frac{15}{28}$$

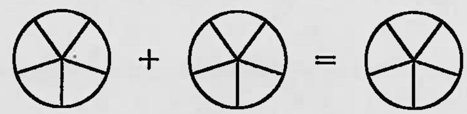
3. **Dividing Fractions.** To divide one fraction by another, you invert the second fraction and then multiply. For example,

$$\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}\times\frac{4}{1}=\frac{1\times 4}{2\times 1}=\frac{4}{2}=\frac{2}{1}=2$$

4. **Adding and Subtracting Fractions.** Fractions must have the same denominator before you can add or subtract them. If the two fractions already have a common denominator, you simply add (or subtract as the case may be) *only* the values in the numerators. For example,

$$\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$$

Suppose you divided a pie into five equal pieces (fifths). If you first ate two-fifths of the pie and then another one-fifth, the total amount eaten would be three-fifths of the pie:



If the two fractions do not have the same denominator, you must first find equivalent fractions with a common denominator before you can add or subtract. The product of the two denominators will always work as a common denominator for equivalent fractions (although it may not be the lowest common denominator). For example,

$$\frac{2}{3} + \frac{1}{10} = ?$$

Because these two fractions have different denominators, it is necessary to convert each into an equivalent fraction and find a common denominator. We will use $3 \times 10 = 30$ as the common denominator. Thus, the equivalent fraction of each is

$$\frac{2}{3} = \frac{20}{30} \quad \text{and} \quad \frac{1}{10} = \frac{3}{30}$$

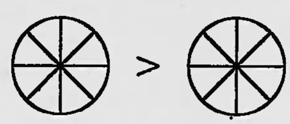
Now the two fractions can be added:

$$\frac{20}{30} + \frac{3}{30} = \frac{23}{30}$$

5. **Comparing the Size of Fractions.** When comparing the size of two fractions with the same denominator, the larger fraction will have the larger numerator. For example,

$$\frac{5}{8} > \frac{3}{8}$$

The denominators are the same, so the whole is partitioned into pieces of the same size. Five of these pieces are more than three of them:



When two fractions have different denominators, you must first convert them to fractions with a common denominator to determine which is larger. Consider the following fractions:

$$\frac{3}{8} \quad \text{and} \quad \frac{7}{16}$$

If the numerator and denominator of $\frac{3}{8}$ are multiplied by 2, the resulting equivalent fraction will have a denominator of 16:

$$\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$$

Now a comparison can be made between the two fractions:

$$\frac{6}{16} < \frac{7}{16}$$

Therefore,

$$\frac{3}{8} < \frac{7}{16}$$

DECIMALS

1. **Converting Decimals to Fractions.** Like a fraction, a decimal represents part of the whole. The first decimal place to the right of the decimal point indicates how many tenths are used. For example,

$$0.1 = \frac{1}{10} \qquad 0.7 = \frac{7}{10}$$

The next decimal place represents $\frac{1}{100}$, the next $\frac{1}{1000}$, the next $\frac{1}{10{,}000}$, and so on. To change a decimal to a fraction, just use the number without the decimal point for the numerator. Use the denominator that the last (on the right) decimal place represents. For example,

$$0.32 = \frac{32}{100} \qquad 0.5333 = \frac{5333}{10{,}000} \qquad 0.05 = \frac{5}{100} \qquad 0.001 = \frac{1}{1000}$$

2. **Adding and Subtracting Decimals.** To add and subtract decimals, the only rule is that you must keep the decimal points in a straight vertical line. For example,

$$\begin{array}{r} 0.27 \\ +\,1.326 \\ \hline 1.526 \end{array} \qquad \begin{array}{r} 3.595 \\ -\,0.67 \\ \hline 2.925 \end{array}$$

3. **Multiplying Decimals.** To multiply two decimal values, you first multiply the two numbers, ignoring the decimal points. Then you position the decimal point in the answer so that the number of digits to the right of the decimal point is equal to the total number of decimal places in the two numbers being multiplied. For example,

$$\begin{array}{rl} 1.73 & \text{(two decimal places)} \\ \times\,0.251 & \text{(three decimal places)} \\ \hline 173 & \\ 865 & \\ 346 & \\ \hline 0.43423 & \text{(five decimal places)} \end{array} \qquad \begin{array}{rl} 0.25 & \text{(two decimal places)} \\ \times\,0.005 & \text{(three decimal places)} \\ \hline 125 & \\ 00 & \\ 00 & \\ \hline 0.00125 & \text{(five decimal places)} \end{array}$$

4. **Dividing Decimals.** The simplest procedure for dividing decimals is based on the fact that dividing two numbers is identical to expressing them as a fraction:

$$0.25 \div 1.6 \text{ is identical to } \frac{0.25}{1.6}$$

You now can multiply both the numerator and the denominator of the fraction by 10, 100, 1000, or whatever number is necessary to remove the decimal places. Remember that multiplying both the numerator and the denominator of a fraction by the *same* value will create an equivalent fraction. Therefore,

$$\frac{0.25}{1.6} = \frac{0.25 \times 100}{1.6 \times 100} = \frac{25}{100} = \frac{5}{32}$$

The result is a division problem without any decimal places in the two numbers.

PERCENTAGES

1. **Converting a Percentage to a Fraction or a Decimal.** To convert a percentage to a fraction, remove the percent sign, place the number in the numerator, and use 100 for the denominator. For example,

$$52\% = \frac{52}{100} \qquad 5\% = \frac{5}{100}$$

To convert a percentage to a decimal, remove the percent sign and divide by 100, or simply move the decimal point two places to the left. For example,

$$83\% = .83. = 0.83$$

$$14.5\% = .14.5 = 0.145$$

$$5\% = .05. = 0.05$$

2. **Performing Arithmetic Operations with Percentages.** There are situations in which it is best to express percent values as decimals in order to perform certain arithmetic operations. For example, what is 45% of 60? This question may be stated as

$$45\% \times 60 = ?$$

The 45% should be converted to decimal form to find the solution to this question. Therefore,

$$0.45 \times 60 = 27$$

ARNING CHECK

1. Convert $\frac{3}{25}$ to a decimal.
2. Convert $\frac{3}{8}$ to a percentage.
3. Next to each set of fractions, write "True" if they are equivalent and "False" if they are not:

   a. $\frac{3}{8} = \frac{9}{24}$ ____________ b. $\frac{7}{9} = \frac{17}{19}$ ____________

   c. $\frac{2}{7} = \frac{4}{14}$ ____________

4. Compute the following:
   a. $\frac{1}{6} \times \frac{7}{10}$ b. $\frac{7}{8} - \frac{1}{2}$ c. $\frac{9}{10} \div \frac{2}{3}$ d. $\frac{7}{22} + \frac{2}{3}$
5. Identify the larger fraction of each pair:
   a. $\frac{7}{10}, \frac{21}{100}$ b. $\frac{3}{4}, \frac{7}{12}$ c. $\frac{22}{3}, \frac{19}{3}$
6. Convert the following decimals into fractions:
   a. 0.012 b. 0.77 c. 0.005
7. $2.59 \times 0.015 = ?$
8. $1.8 \div 0.02 = ?$
9. What is 28% of 45?

ANSWERS 1. 0.12 2. 37.5% 3. a. True b. False c. True
4. a. $\frac{7}{60}$ b. $\frac{3}{8}$ c. $\frac{27}{20}$ d. $\frac{65}{66}$ 5. a. $\frac{7}{10}$ b. $\frac{3}{4}$ c. $\frac{22}{3}$
6. a. $\frac{12}{1000} = \frac{3}{250}$ b. $\frac{77}{100}$ c. $\frac{5}{1000} = \frac{1}{200}$ 7. 0.03885 8. 90 9. 12.6

A.3 NEGATIVE NUMBERS

Negative numbers are used to represent values less than zero. Negative numbers may occur when you are measuring the difference between two scores. For example, a researcher may want to evaluate the effectiveness of a propaganda film by measuring people's attitudes with a test both before and after viewing the film:

	Before	*After*	*Amount of Change*
Person A	23	27	+4
Person B	18	15	−3
Person C	21	16	−5

Notice that the negative sign provides information about the direction of the difference: A plus sign indicates an increase in value, and a minus sign indicates a decrease.

Because negative numbers are frequently encountered, you should be comfortable working with these values. This section reviews basic arithmetic operations using negative numbers. You should also note that any number without a sign (+ or −) is assumed to be positive.

1. **Adding Negative Numbers.** When adding numbers that include negative values, simply interpret the negative sign as subtraction. For example,

$$3 + (-2) + 5 = 3 - 2 + 5 = 6$$

When adding a long string of numbers, it often is easier to add all of the positive values to obtain the positive sum and then to add all of the negative values to obtain the negative sum. Finally, you subtract the negative sum from the positive sum. For example,

$$-1 + 3 + (-4) + 3 + (-6) + (-2)$$

positive sum = 6 negative sum = 13

Answer: $6 - 13 = -7$

2. **Subtracting Negative Numbers.** To subtract a negative number, change it to a positive number, and add. For example,

$$4 - (-3) = 4 + 3 = 7$$

This rule is easier to understand if you think of positive numbers as financial gains and negative numbers as financial losses. In this context, taking away a debt is equivalent to a financial gain. In mathematical terms, taking away a negative number is equivalent to adding a positive number. For example, suppose you are meeting a friend for lunch. You have \$7, but you owe your friend \$3. Thus, you really have only \$4 to spend for lunch. But your friend forgives (takes away) the \$3 debt. The result is that you now have \$7 to spend. Expressed as an equation,

\$4 minus a \$3 debt = \$7

$$4 - (-3) = 4 + 3 = 7$$

3. **Multiplying and Dividing Negative Numbers.** When the two numbers being multiplied (or divided) have the same sign, the result is a positive number. When the two numbers have different signs, the result is negative. For example,

$$3 \times (-2) = -6$$

$$-4 \times (-2) = +8$$

The first example is easy to explain by thinking of multiplication as repeated addition. In this case,

$$3 \times (-2) = (-2) + (-2) + (-2) = -6$$

You add three negative 2s, which results in a total of negative 6. In the second example, we are multiplying by a negative number. This amounts to repeated subtraction. That is,

$$\begin{aligned} -4 \times (-2) &= -(-2) - (-2) - (-2) - (-2) \\ &= 2 + 2 + 2 + 2 = 8 \end{aligned}$$

By using the same rule for both multiplication and division, we ensure that these two operations are compatible. For example,

$$-6 \div 3 = -2$$

which is compatible with

$$3 \times (-2) = -6$$

Also,

$$8 \div (-4) = -2$$

which is compatible with

$$-4 \times (-2) = +8$$

LEARNING CHECK

1. Complete the following calculations:
   a. $3 + (-8) + 5 + 7 + (-1) + (-3)$
   b. $5 - (-9) + 2 - (-3) - (-1)$
   c. $3 - 7 - (-21) + (-5) - (-9)$
   d. $4 - (-6) - 3 + 11 - 14$
   e. $9 + 8 - 2 - 1 - (-6)$
   f. $9 \times (-3)$
   g. $-7 \times (-4)$
   h. $-6 \times (-2) \times (-3)$
   i. $-12 \div (-3)$
   j. $18 \div (-6)$

ANSWERS 1. a. 3 b. 20 c. 21 d. 4 e. 20
f. −27 g. 28 h. −36 i. 4 j. −3

A.4 BASIC ALGEBRA: SOLVING EQUATIONS

An equation is a mathematical statement that indicates two quantities are identical. For example,

$$12 = 8 + 4$$

Often an equation will contain an unknown (or variable) quantity that is identified with a letter or symbol, rather than a number. For example,

$$12 = 8 + X$$

In this event, your task is to find the value of X that makes the equation "true," or balanced. For this example, an X value of 4 will make a true equation. Finding the value of X is usually called *solving the equation.*

To solve an equation, there are two points to keep in mind:

1. Your goal is to have the unknown value (X) isolated on one side of the equation. This means that you need to remove all of the other numbers and symbols that appear on the same side of the equation as the X.
2. The equation remains balanced, provided that you treat both sides exactly the same. For example, you could add 10 points to *both* sides, and the solution (the X value) for the equation would be unchanged.

NDING THE SOLUTION FOR AN EQUATION

We will consider four basic types of equations and the operations needed to solve them.

1. **When X Has a Value Added to It.** An example of this type of equation is

$$X + 3 = 7$$

Your goal is to isolate X on one side of the equation. Thus, you must remove the +3 on the left-hand side. The solution is obtained by subtracting 3 from *both* sides of the equation:

$$X + 3 - 3 = 7 - 3$$
$$X = 4$$

The solution is $X = 4$. You should always check your solution by returning to the original equation and replacing X with the value you obtained for the solution. For this example,

$$X + 3 = 7$$
$$4 + 3 = 7$$
$$7 = 7$$

2. When X Has a Value Subtracted From It. An example of this type of equation is

$$X - 8 = 12$$

In this example, you must remove the −8 from the left-hand side. Thus, the solution is obtained by adding 8 to *both* sides of the equation:

$$X - 8 + 8 = 12 + 8$$
$$X = 20$$

Check the solution:

$$X - 8 = 12$$
$$20 - 8 = 12$$
$$12 = 12$$

3. When X Is Multiplied by a Value. An example of this type of equation is

$$4X = 24$$

In this instance, it is necessary to remove the 4 that is multiplied by X. This may be accomplished by dividing both sides of the equation by 4:

$$\frac{4X}{4} = \frac{24}{4}$$
$$X = 6$$

Check the solution:

$$4X = 24$$
$$4(6) = 24$$
$$24 = 24$$

4. When X Is Divided by a Value. An example of this type of equation is

$$\frac{X}{3} = 9$$

Now the X is divided by 3, so the solution is obtained by multiplying by 3. Multiplying both sides yields

$$3\left(\frac{X}{3}\right) = 9(3)$$

$$X = 27$$

For the check,

$$\frac{X}{3} = 9$$

$$\frac{27}{3} = 9$$

$$9 = 9$$

SOLUTIONS FOR MORE COMPLEX EQUATIONS

More complex equations can be solved by using a combination of the preceding simple operations. Remember that at each stage you are trying to isolate X on one side of the equation. For example,

$$3X + 7 = 22$$

$$3X + 7 - 7 = 22 - 7 \quad \text{(Remove +7 by subtracting 7 from both sides.)}$$

$$3X = 15$$

$$\frac{3X}{3} = \frac{15}{3} \quad \text{(Remove 3 by dividing both sides by 3.)}$$

$$X = 5$$

To check this solution, return to the original equation, and substitute 5 in place of X:

$$3X + 7 = 22$$

$$3(5) + 7 = 22$$

$$15 + 7 = 22$$

$$22 = 22$$

Following is another type of complex equation frequently encountered in statistics:

$$\frac{X+3}{4} = 2$$

First, remove the 4 by multiplying both sides by 4:

$$4\left(\frac{X+3}{4}\right) = 2(4)$$

$$X + 3 = 8$$

Now remove the +3 by subtracting 3 from both sides:

$$X + 3 - 3 = 8 - 3$$

$$X = 5$$

To check this solution, return to the original equation, and substitute 5 in place of X:

$$\frac{X+3}{4}=2$$

$$\frac{5+3}{4}=2$$

$$\frac{8}{4}=2$$

$$2=2$$

LEARNING CHECK

1. Solve for X, and check the solutions:

   a. $3X = 18$ b. $X + 7 = 9$ c. $X - 4 = 18$ d. $5X - 8 = 12$

   e. $\frac{X}{9} = 5$ f. $\frac{X+1}{6} = 4$ g. $X + 2 = -5$ h. $\frac{X}{5} = -5$

   i. $\frac{2X}{3} = 12$ j. $\frac{X}{3} + 1 = 3$

ANSWERS 1. a. $X = 6$ b. $X = 2$ c. $X = 22$ d. $X = 4$ e. $X = 45$ f. $X = 23$ g. $X = -7$ h. $X = -25$ i. $X = 18$ j. $X = 6$

A.5 EXPONENTS AND SQUARE ROOTS

EXPONENTIAL NOTATION

A simplified notation is used whenever a number is being multiplied by itself. The notation consists of placing a value, called an *exponent*, on the right-hand side of and raised above another number, called a *base*. For example,

$$7^3 \leftarrow \text{exponent}$$
↑
base

The exponent indicates how many times the base is used as a factor in multiplication. Following are some examples:

$7^3 = 7(7)(7)$ (Read "7 cubed" or "7 raised to the third power")
$5^2 = 5(5)$ (Read "5 squared")
$2^5 = 2(2)(2)(2)(2)$ (Read "2 raised to the fifth power")

There are a few basic rules about exponents that you will need to know for this course. They are outlined here.

1. **Numbers Raised to One or Zero.** Any number raised to the first power equals itself. For example,

   $6^1 = 6$

Any number (except zero) raised to the zero power equals 1. For example,

$$9^0 = 1$$

2. **Exponents for Multiple Terms.** The exponent applies only to the base that is just in front of it. For example,

$$XY^2 = XYY$$
$$a^2b^3 = aabbb$$

3. **Negative Bases Raised to an Exponent.** If a negative number is raised to a power, then the result will be positive for exponents that are even and negative for exponents that are odd. For example,

$$\begin{aligned}(-4)^3 &= -4(-4)(-4)\\ &= 16(-4)\\ &= -64\end{aligned}$$

and

$$\begin{aligned}(-3)^4 &= -3(-3)(-3)(-3)\\ &= 9(-3)(-3)\\ &= 9(9)\\ &= 81\end{aligned}$$

Note: The parentheses are used to ensure that the exponent applies to the entire negative number, including the sign. Without the parentheses there is some ambiguity as to how the exponent should be applied. For example, the expression -3^2 could have two interpretations:

$$-3^2 = (-3)(-3) = 9 \quad \text{or} \quad -3^2 = -(3)(3) = -9$$

4. **Exponents and Parentheses.** If an exponent is present outside of parentheses, then the computations within the parentheses are done first, and the exponential computation is done last:

$$(3 + 5)^2 = 8^2 = 64$$

Notice that the meaning of the expression is changed when each term in the parentheses is raised to the exponent individually:

$$3^2 + 5^2 = 9 + 25 = 34$$

Therefore,

$$X^2 + Y^2 \neq (X + Y)^2$$

5. **Fractions Raised to a Power.** If the numerator and denominator of a fraction are each raised to the same exponent, then the entire fraction can be raised to that exponent. That is,

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

For example,

$$\frac{3^2}{4^2} = \left(\frac{3}{4}\right)^2$$

$$\frac{9}{16} = \frac{3}{4}\left(\frac{3}{4}\right)$$

$$\frac{9}{16} = \frac{9}{16}$$

SQUARE ROOTS

The square root of a value equals a number that, when multiplied by itself, yields the original value. For example, the square root of 16 equals 4, because 4 times 4 equals 16. The symbol for the square root is called a *radical*, $\sqrt{\;}$. The square root is taken for the number under the radical. For example,

$$\sqrt{16} = 4$$

Finding the square root is the inverse of raising a number to the second power (squaring). Thus,

$$\sqrt{a^2} = a$$

For example,

$$\sqrt{3^2} = \sqrt{9} = 3$$

Also,

$$\left(\sqrt{b}\right)^2 = b$$

For example,

$$\left(\sqrt{64}\right)^2 = 8^2 = 64$$

Computations under the same radical are performed *before* the square root is taken. For example,

$$\sqrt{9 + 16} = \sqrt{25} = 5$$

Note that with addition (or subtraction), separate radicals yield a different result:

$$\sqrt{9} + \sqrt{16} = 3 + 4 = 7$$

Therefore,

$$\sqrt{X} + \sqrt{Y} \neq \sqrt{X + Y}$$
$$\sqrt{X} - \sqrt{Y} \neq \sqrt{X - Y}$$

If the numerator and denominator of a fraction each have a radical, then the entire fraction can be placed under a single radical:

$$\frac{\sqrt{16}}{\sqrt{4}} = \sqrt{\frac{16}{4}}$$

$$\frac{4}{2} = \sqrt{4}$$

$$2 = 2$$

Therefore,

$$\frac{\sqrt{X}}{\sqrt{Y}} = \sqrt{\frac{X}{Y}}$$

Also, if the square root of one number is multiplied by the square root of another number, then the same result would be obtained by taking the square root of the product of both numbers. For example,

$$\sqrt{9} \times \sqrt{16} = \sqrt{9 \times 16}$$
$$3 \times 4 = \sqrt{144}$$
$$12 = 12$$

Therefore,

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

LEARNING CHECK

1. Perform the following computations:
   a. $(-6)^3$
   b. $(3 + 7)^2$
   c. a^3b^2 when $a = 2$ and $b = -5$
   d. a^4b^3 when $a = 2$ and $b = 3$
   e. $(XY)^2$ when $X = 3$ and $Y = 5$
   f. $X^2 + Y^2$ when $X = 3$ and $Y = 5$
   g. $(X + Y)^2$ when $X = 3$ and $Y = 5$
   h. $\sqrt{5+4}$
   i. $\left(\sqrt{9}\right)^2$
   j. $\frac{\sqrt{16}}{\sqrt{4}}$

ANSWERS 1. a. −216 b. 100 c. 200 d. 432 e. 225
f. 34 g. 64 h. 3 i. 9 j. 2

PROBLEMS FOR APPENDIX A Basic Mathematics Review

1. $50/(10 - 8) = ?$
2. $(2 + 3)^2 = ?$
3. $20/10 \times 3 = ?$
4. $12 - 4 \times 2 + 6/3 = ?$
5. $24/(12 - 4) + 2 \times (6 + 3) = ?$
6. Convert $\frac{7}{20}$ to a decimal.
7. Express $\frac{9}{25}$ as a percentage.
8. Convert 0.91 to a fraction.
9. Express 0.0031 as a fraction.
10. Next to each set of fractions, write "True" if they are equivalent and "False" if they are not:
    a. $\frac{4}{1000} = \frac{2}{100}$ ____________
    b. $\frac{5}{6} = \frac{52}{62}$ ____________
    c. $\frac{1}{8} = \frac{7}{56}$ ____________
11. Perform the following calculations:
    a. $\frac{4}{5} \times \frac{2}{3} = ?$ b. $\frac{7}{9} \div \frac{2}{3} = ?$
    c. $\frac{3}{8} + \frac{1}{5} = ?$ d. $\frac{5}{18} - \frac{1}{6} = ?$
12. $2.51 \times 0.017 = ?$
13. $3.88 \times 0.0002 = ?$
14. $3.17 + 17.0132 = ?$
15. $5.55 + 10.7 + 0.711 + 3.33 + 0.031 = ?$
16. $2.04 \div 0.2 = ?$
17. $0.36 \div 0.4 = ?$
18. $5 + 3 - 6 - 4 + 3 = ?$
19. $9 - (-1) - 17 + 3 - (-4) + 5 = ?$
20. $5 + 3 - (-8) - (-1) + (-3) - 4 + 10 = ?$
21. $8 \times (-3) = ?$
22. $-22 \div (-2) = ?$
23. $-2(-4)\_(-3) = ?$
24. $84 \div (-4) = ?$

Solve the equations in problems 25–32 for X.

25. $X - 7 = -2$
26. $9 = X + 3$
27. $\frac{X}{4} = 11$
28. $-3 = \frac{X}{3}$
29. $\frac{X + 3}{5} = 2$
30. $\frac{X + 1}{3} = -8$
31. $6X - 1 = 11$
32. $2X + 3 = -11$
33. $(-5)^2 = ?$
34. $(-5)^3 = ?$
35. If $a = 4$ and $b = 3$, then $a^2 + b^4 = ?$
36. If $a = -1$ and $b = 4$, then $(a + b)^2 = ?$
37. If $a = -1$ and $b = 5$, then $ab^2 = ?$
38. $\frac{8}{\sqrt{4}} = ?$
39. $\sqrt{\frac{20}{5}} = ?$

SKILLS ASSESSMENT FINAL EXAM

SECTION 1

1. $4 + 8/4 = ?$
2. $(4 + 8)/4 = ?$
3. $4 \times 3^2 = ?$
4. $(4 \times 3)^2 = ?$
5. $10/5 \times 2 = ?$
6. $10/(5 \times 2) = ?$
7. $40 - 10 \times 4/2 = ?$
8. $(5 - 1)^2/2 = ?$
9. $3 \times 6 - 3^2 = ?$
10. $2 \times (6 - 3)^2 = ?$
11. $4 \times 3 - 1 + 8 \times 2 = ?$
12. $4 \times (3 - 1 + 8) \times 2 = ?$

SECTION 2

1. Express $\frac{14}{80}$ as a decimal.
2. Convert $\frac{6}{25}$ to a percentage.
3. Convert 18% to a fraction.
4. $\frac{3}{5} \times \frac{2}{3} = ?$
5. $\frac{5}{24} + \frac{5}{6} = ?$
6. $\frac{7}{12} \div \frac{5}{6} = ?$
7. $\frac{5}{9} - \frac{1}{3} = ?$
8. $6.11 \times 0.22 = ?$
9. $0.18 \div 0.9 = ?$
10. $8.742 + 0.76 = ?$
11. In a statistics class of 72 students, three-eighths of the students received a B on the first test. How many Bs were earned?
12. What is 15% of 64?

SECTION 3

1. $3 - 1 - 3 + 5 - 2 + 6 = ?$
2. $-8 - (-6) = ?$
3. $2 - (-7) - 3 + (-11) - 20 = ?$
4. $-8 - 3 - (-1) - 2 - 1 = ?$
5. $8(-2) = ?$
6. $-7(-7) = ?$
7. $-3(-2)(-5) = ?$
8. $-3(5)(-3) = ?$
9. $-24 \div (-4) = ?$
10. $36 \div (-6) = ?$
11. $-56/7 = ?$
12. $-7/(-1) = ?$

SECTION 4

Solve for *X*.

1. $X + 5 = 12$
2. $X - 11 = 3$
3. $10 = X + 4$
4. $4X = 20$
5. $\frac{X}{2} = 15$
6. $18 = 9X$
7. $\frac{X}{5} = 35$
8. $2X + 8 = 4$
9. $\frac{X+1}{3} = 6$
10. $4X + 3 = -13$
11. $\frac{X+3}{3} = -7$
12. $23 = 2X - 5$

SECTION 5

1. $5^3 = ?$
2. $(-4)^3 = ?$
3. $(-2)^5 = ?$
4. $(-2)^6 = ?$
5. If $a = 4$ and $b = 2$, then $ab^2 = ?$
6. If $a = 4$ and $b = 2$, then $(a + b)^3 = ?$
7. If $a = 4$ and $b = 2$, then $a^2 + b^2 = ?$
8. $(11 + 4)^2 = ?$
9. $\sqrt{7^2} = ?$
10. If $a = 36$ and $b = 64$, then $\sqrt{a + b} = ?$
11. $\frac{25}{\sqrt{25}} = ? = ?$
12. If $a = -1$ and $b = 2$, then $a^3b^4 = ?$

ANSWER KEY Skills Assessment Exams

PREVIEW EXAM

SECTION 1

1. 17	2. 35	3. 6
4. 24	5. 5	6. 2
7. $\frac{1}{3}$	8. 8	9. 72
10. 8	11. 24	12. 48

SECTION 2

1. 75%	2. $\frac{30}{100}$, or $\frac{3}{10}$	3. 0.3
4. $\frac{10}{13}$	5. 1.625	6. $\frac{2}{20}$, or $\frac{1}{10}$
7. $\frac{19}{24}$	8. 1.4	9. $\frac{4}{15}$
10. 7.5	11. 16	12. 36

SECTION 3

1. 4	2. 8	3. 2
4. 9	5. −12	6. 12
7. −15	8. −24	9. −4
10. 3	11. −2	12. 25

FINAL EXAM

SECTION 1

1. 6	2. 3	3. 36
4. 144	5. 4	6. 1
7. 20	8. 8	9. 9
10. 18	11. 27	12. 80

SECTION 2

1. 0.175	2. 24%	3. $\frac{18}{100}$, or $\frac{9}{50}$
4. $\frac{6}{15}$, or $\frac{2}{5}$	5. $\frac{25}{24}$	6. $\frac{42}{60}$, or $\frac{7}{10}$
7. $\frac{2}{9}$	8. 1.3442	9. 0.2
10. 9.502	11. 27	12. 9.6

SECTION 3

1. 8	2. −2	3. −25
4. −13	5. −16	6. 49
7. −30	8. 45	9. 6
10. −6	11. −8	12. 7

PREVIEW EXAM

SECTION 4

1. $X = 7$	2. $X = 29$	3. $X = 9$
4. $X = 4$	5. $X = 24$	6. $X = 15$
7. $X = 80$	8. $X = -3$	9. $X = 11$
10. $X = 25$	11. $X = 11$	12. $X = 7$

SECTION 5

1. 64	2. 4	3. 54
4. 25	5. 13	6. −27
7. 256	8. 8	9. 12
10. 121	11. 33	12. −9

FINAL EXAM

SECTION 4

1. $X = 7$	2. $X = 14$	3. $X = 6$
4. $X = 5$	5. $X = 30$	6. $X = 2$
7. $X = 175$	8. $X = -2$	9. $X = 17$
10. $X = -4$	11. $X = -24$	12. $X = 14$

SECTION 5

1. 125	2. −64	3. −32
4. 64	5. 16	6. 216
7. 20	8. 225	9. 7
10. 10	11. 5	12. −16

SOLUTIONS TO SELECTED PROBLEMS FOR APPENDIX A Basic Mathematics Review

1. 25
5. 21
7. 36%
10. b. False
11. a. $\frac{8}{15}$ b. $\frac{21}{18}$ c. $\frac{23}{40}$
12. 0.04267
3. 6
6. 0.35
9. $\frac{31}{10,000}$
14. 20.1832
17. 0.9
21. −24
25. $X = 5$
30. $X = -25$
34. −125
37. −25
19. 5
22. 11
28. $X = -9$
31. $X = 2$
36. 9
39. 2

SUGGESTED REVIEW BOOKS

There are many basic mathematics books available if you need a more extensive review than this appendix can provide. Several are probably available in your library. The following books are but a few of the many that you may find helpful:

Gustafson, R. D., Karr, R., & Massey, M. (2011). *Beginning Algebra* (9th ed.). Belmont, CA: Brooks/Cole.

Lial, M. L., Salzman, S.A., & Hestwood, D.L. (2010). *Basic College Mathematics* (8th ed). Reading MA: Addison-Wesley.

McKeague, C. P. (2010). *Basic College Mathematics: A Text/Workbook.* (7th ed.). Belmont, CA: Brooks/Cole.