$$\text{Fit} = \text{Cell Avg} + \text{Block effect}$$
$$(56) \quad + \quad (3) \quad = 59$$
$$\text{Res} = \text{Obs} - \text{Fit}$$
$$(57) - (59) = -2$$

b. Degrees of freedom and ANOVA

$df_{Grand} = 1$

$df_{Operations} = 1$ There are two effects, which add to zero.

$df_{Dogs} = 8$ Each group of five dogs has five dog effects that add to zero in the group: 4 df from each of 2 groups.

$df_{Methods} = 1$ There are two effects, which add to zero.

$df_{Inter} = 1$ There are 4 effects, in a 2×2 rectangle. They add to zero across rows and down columns, leaving 1×1 free numbers.

$df_{Res} = 8$ By subtraction, using $df_{Total} = 20$.

In constructing the ANOVA table, we use MS_{Dogs} in the denominator of the *F*-ratio for Operations, the between-blocks factor, and we use MS_{Res} in the denominator of the other *F*-ratios.

Source	SS	df	MS	*F*-ratio	Crit.val.
Grand Average	30420	1	30420		
Operations (betw.)	320	1	320.0	4.96	5.32
Dogs (blocks)	508	8	63.5	0.75	3.44
Methods (w/in)	2420	1	2420.0	28.47*	5.32
Interaction	80	1	80.0	0.94	5.32
Residual	680	8	85.0		
Total	34428	20			

From the ANOVA table we conclude that overall differences due to operation are not quite big enough to be declared "real," but there is a detectable difference between the two methods. The other observed differences, however, for dogs and interaction, are roughly the same size as the residual errors and could easily be due just to chance variation. ■

Two kinds of units, two kinds of chance error. For the SP/RM design there is an exception to the intuitive principle that you should use the mean square for residuals in the denominator of your *F*-ratios. Chapter 14 discusses the logic of choosing these denominators and gives a single rule that works for all balanced designs. For now, however, I'll give an informal account of the logic behind the *F*-ratios for SP/RM designs:

The SP/RM has two kinds of error variability:

- whole-plot error comes from differences among the larger units;

+ Block effect
+ (3) = 59

9) = −2

d ANOVA

e are two effects, which add to zero.
group of five dogs has five dog effects that add to zero in
group: 4 df from each of 2 groups.
re are two effects, which add to zero.
re are 4 effects, in a 2 × 2 rectangle. They add to zero
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ne ANOVA table, we use MS_{Dogs} in the denominator of the
ns, the between-blocks factor, and we use MS_{Res} in the de-
ner F-ratios.

SS	df	MS	F-ratio	Crit.val.
30420	1	30420		
320	1	320.0	4.96	5.32
508	8	63.5	0.75	3.44
2420	1	2420.0	28.47*	5.32
80	1	80.0	0.94	5.32
680	8	85.0		
34428	20			

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es a single rule that works for all balanced designs. For now, how-
es a single rule that works for all balanced designs:
mal account of the logic behind the F-ratios for SP/RM designs:

as two kinds of error variability:
error comes from differences among the larger units;
... the smaller units.

EXAMPLE 7.17 AN ANALOGY: KIDS ON SHIPS

(This analogy asks you to visualize error variability as up-and-down motion.) Think of a fleet of ships—the larger units—riding up and down on swells of the ocean. This up-and-down motion of the ships is the variability associated with the larger units—the "whole-plot error."

Next imagine bunches of children—the smaller units—jumping up and down on the decks of the ships. Their motion is the variability associated with the smaller units—the "subplot error."

Now for the treatments. Suppose we load the ships with various amounts of cargo—we apply this (whole-plot) treatment to the ships, not to the children. But we might also outfit the jumping children with different brands of tennis shoes—we apply this (subplot) treatment to the children, not to the ships.

The virtue of this example (if there is one!) is that it reduces the logic of the F-tests to common sense: to judge the effects of the cargo (between-blocks factor), you would compare ships (blocks). Judging cargo by comparing children is obviously wrong. Similarly, to judge the effects of the shoes, you'd compare children; judging shoes by comparing ships would be ridiculous. ■

Between-blocks treatments get assigned to the larger units, blocks, or whole plots; in a sense what goes on in the subplots is irrelevant to testing the between-blocks factor. In thinking about F-ratios for between-blocks factors, it is useful to think of each block as contributing just one response value, the block average, and to ask yourself how you would analyze the resulting set of averages. If the pattern that assigns treatments to blocks is CR (as it is for all the SP/RM designs so far), then for these treatments MS_{Blocks} plays the same role as MS_{Res} for the CR design.

EXAMPLE 7.18 DIABETIC DOGS: THE BETWEEN-BLOCKS F-RATIO

I'll use the data from Example 7.16 to illustrate why you use MS_{Blocks} in the denominator of the F-ratio for the between-blocks factor. For these data the between-blocks factor—Operations—was assigned to Dogs (blocks) completely at random, as in a CR design. If you think of each block average as a single response value for its block, the resulting set of 10 block averages has the same structure as a CR[1] (Fig. 7.21).

	Con	Dia		Con	Dia		Con	Dia		Con	Dia
	36	48		39	39		−4	4		1	5
	28	33		39	39		−4	4		−7	−10
	36	39	=	39	39	+	−4	4	+	1	−4
	39	49		39	39		−4	4		4	6
	36	46		39	39		−4	4		1	3
	Block Avg			Grand Avg			Betw eff			Block eff	
SS	1935		=	1521		+	160		+	254	
DF	10		=	1		+	1		+	8	